DefElement
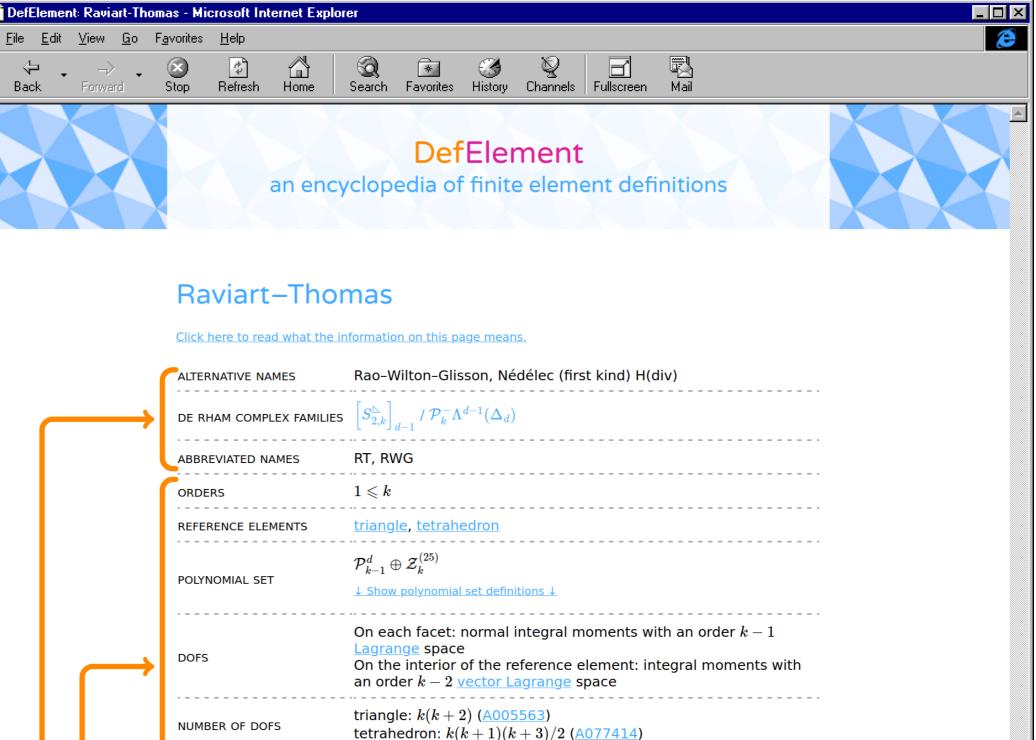## an encyclopedia of finite element definitions

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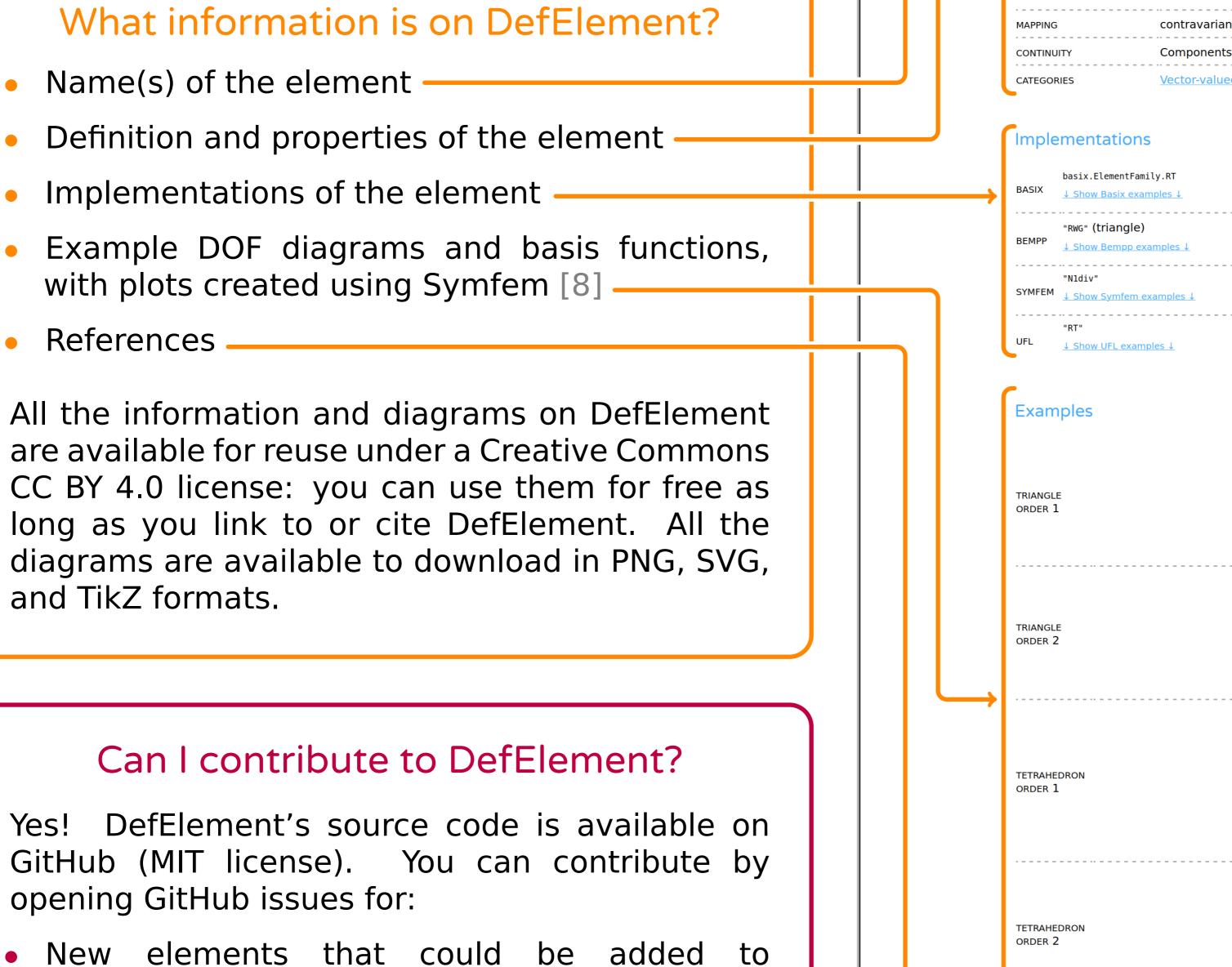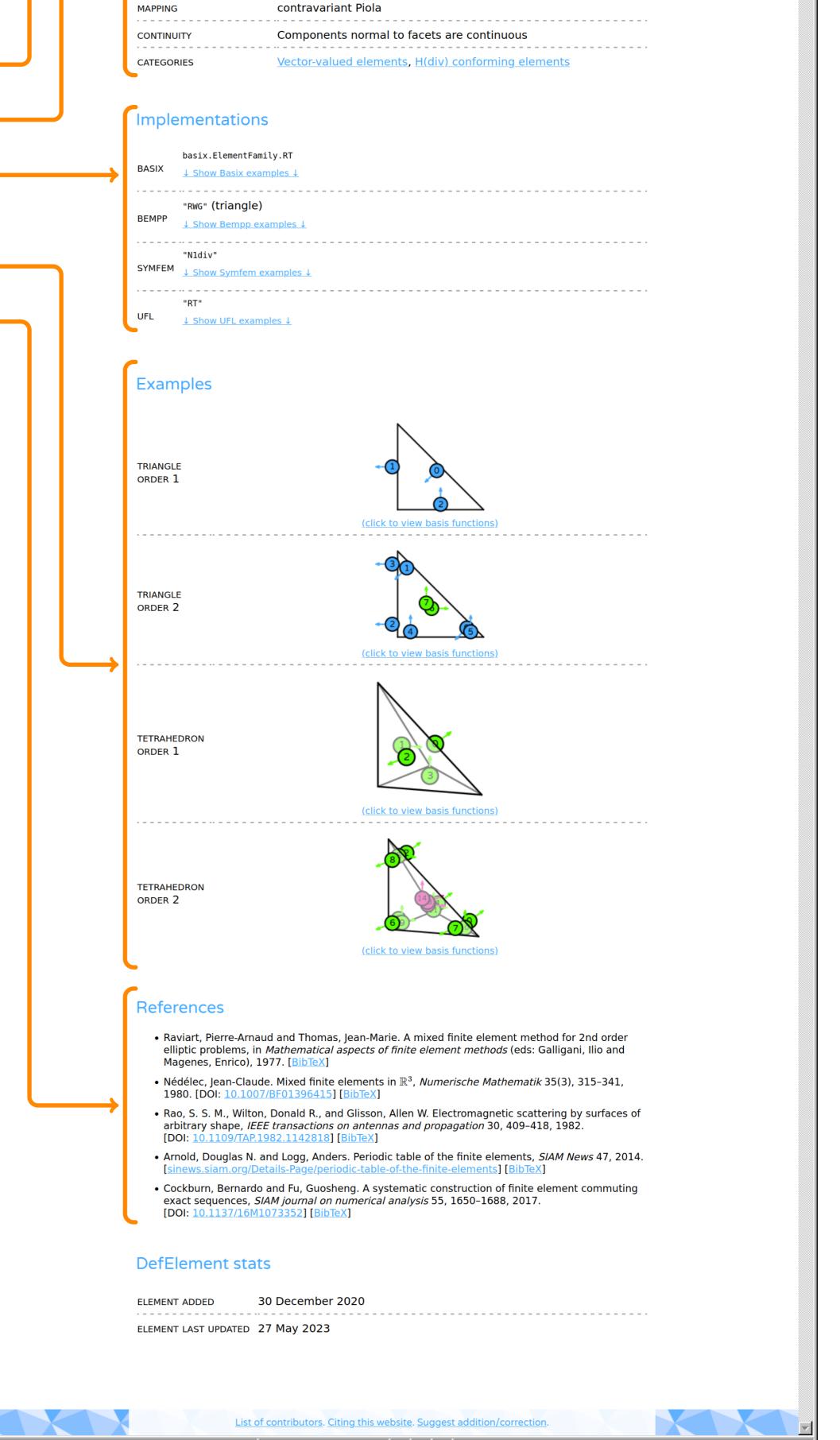
What is DefElement?

DefElement is an online encyclopedia of finite element definitions. You can view it at

defelement.com

DefElement includes definitions of a huge range of finite elements including commonly used elements such as Lagrange, Raviart–Thomas [6], and Nédélec [4, 5]; and more exotic elements such as Argyris [1], Regge [7, 2], and TNT [3].

What information is on DefElement?

- Name(s) of the element
- Definition and properties of the element
- Implementations of the element
- Example DOF diagrams and basis functions, with plots created using Symfem [8]
- References

All the information and diagrams on DefElement are available for reuse under a Creative Commons CC BY 4.0 license: you can use them for free as long as you link to or cite DefElement. All the diagrams are available to download in PNG, SVG, and TikZ formats.

Can I contribute to DefElement?

Yes! DefElement's source code is available on GitHub (MIT license). You can contribute by opening GitHub issues for:

- New elements that could be added to DefElement.
- Any improvements that you want to suggest.
- Any mistakes that you find.

Or, you could fork the repository and open a pull request to:

- Add implementation information for a finite element library that you use or maintain.
- Resolve any of the currently open issues: keep an eye out for anything tagged *good first issue*.
- Anything else you want to suggest changing.

Browser window — DefElement: Raviart–Thomas page

Raviart–Thomas

Click here to read what the information on this page means.

ALTERNATIVE NAMES	Rao–Wilton–Glisson, Nédélec (first kind) H(div)
DE RHAM COMPLEX FAMILIES	$\left[S_{2,k}^{\triangle}\right]_{d-1}$ / $\mathcal{P}_k^-\Lambda^{d-1}(\Delta_d)$
ABBREVIATED NAMES	RT, RWG
ORDERS	$1 \leqslant k$
REFERENCE ELEMENTS	triangle, tetrahedron
POLYNOMIAL SET	$\mathcal{P}_{k-1}^d \oplus \mathcal{Z}_k^{(25)}$ ↓ Show polynomial set definitions ↓
DOFS	On each facet: normal integral moments with an order $k-1$ Lagrange space On the interior of the reference element: integral moments with an order $k-2$ vector Lagrange space
NUMBER OF DOFS	triangle: $k(k+2)$ (A005563) tetrahedron: $k(k+1)(k+3)/2$ (A077414)
MAPPING	contravariant Piola
CONTINUITY	Components normal to facets are continuous
CATEGORIES	Vector-valued elements, H(div) conforming elements

Implementations

BASIX	`basix.ElementFamily.RT` ↓ Show Basix examples ↓
BEMPP	`"RWG"` (triangle) ↓ Show Bempp examples ↓
SYMFEM	`"N1div"` ↓ Show Symfem examples ↓
UFL	`"RT"` ↓ Show UFL examples ↓

Examples

TRIANGLE ORDER 1	(click to view basis functions)
TRIANGLE ORDER 2	(click to view basis functions)
TETRAHEDRON ORDER 1	(click to view basis functions)
TETRAHEDRON ORDER 2	(click to view basis functions)

References

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DefElement stats

ELEMENT ADDED	30 December 2020
ELEMENT LAST UPDATED	27 May 2023

List of contributors. Citing this website. Suggest addition/correction.

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[5] Jean-Claude Nédélec. A new family of mixed finite elements in $\mathbb{R}^3$. *Numerische Mathematik*, 50(1):57–81, 1986.

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[7] Tullio Regge. General relativity without coordinates. *Il Nuovo Cimento*, 19(3):558–571, 1961.

[8] Matthew W. Scroggs. Symfem: a symbolic finite element definition library. *Journal of Open Source Software*, 6(64):3556, 2021.