

# DefElement

an encyclopedia of finite element definitions

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## What is DefElement?

DefElement is an online encyclopedia of finite element definitions. You can view it at

[defelement.com](http://defelement.com)

DefElement includes definitions of a huge range of finite elements including commonly used elements such as Lagrange, Raviart–Thomas [6], and Nédélec [4, 5]; and more exotic elements such as Argyris [1], Regge [7, 2], and TNT [3].

## What information is on DefElement?

- Name(s) of the element
- Definition and properties of the element
- Implementations of the element
- Example DOF diagrams and basis functions, with plots created using Symfem [8]
- References

All the information and diagrams on DefElement are available for reuse under a Creative Commons CC BY 4.0 license: you can use them for free as long as you link to or cite DefElement. All the diagrams are available to download in PNG, SVG, and TikZ formats.

## Can I contribute to DefElement?

Yes! DefElement's source code is available on GitHub (MIT license). You can contribute by opening GitHub issues for:

- New elements that could be added to DefElement.
- Any improvements that you want to suggest.
- Any mistakes that you find.

Or, you could fork the repository and open a pull request to:

- Add implementation information for a finite element library that you use or maintain.
- Resolve any of the currently open issues: keep an eye out for anything tagged *good first issue*.
- Anything else you want to suggest changing.

**Raviart–Thomas**

Click here to read what the information on this page means.

ALTERNATIVE NAMES Rao-Wilton–Glisson, Nédélec (first kind) H(div)

DE RHAM COMPLEX FAMILIES  $\{S_{2k}^1\}_{k \geq 1} / \mathcal{P}_k \wedge^{k+1}(\Delta_2)$

ABBREVIATED NAMES RT, RWG

ORDERS  $1 \leq k$

REFERENCE ELEMENTS triangle, tetrahedron

POLYNOMIAL SET  $\mathcal{P}_{k+1}^d \oplus \mathcal{Z}_k^{(2)}$   
[Show polynomial set definitions ↓](#)

DOFS On each facet: normal integral moments with an order  $k - 1$  Lagrange space  
 On the interior of the reference element: integral moments with an order  $k - 2$  vector Lagrange space

NUMBER OF DOFS triangle:  $k(k + 2)$  (A005563)  
 tetrahedron:  $k(k + 1)(k + 3)/2$  (A077414)

MAPPING contravariant Piola

CONTINUITY Components normal to facets are continuous

CATEGORIES [Vector-valued elements, H\(div\) conforming elements](#)

**Implementations**

BASIX [basix.ElementFamily\\_RT](#)  
[Show Basix examples ↓](#)

BEMPP ["rw" \(triangle\)](#)  
[Show BEMPP examples ↓](#)

SYMFEM ["Ndiv"](#)  
[Show Symfem examples ↓](#)

UFL ["RT"](#)  
[Show UFL examples ↓](#)

**Examples**

TRIANGLE ORDER 1   
[click to view basis functions](#)

TRIANGLE ORDER 2   
[click to view basis functions](#)

TETRAHEDRON ORDER 1   
[click to view basis functions](#)

TETRAHEDRON ORDER 2   
[click to view basis functions](#)

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**DefElement stats**

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## References

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- [7] Silvio Regge. General relativity without coordinates. *Il Nuovo Cimento*, 18(7):558–571, 1961.
- [8] Matthew W. Scroggs. Symfem: A symbolic finite element definition library. *Journal of Open Source Software*, 6(6):0556, 2021.